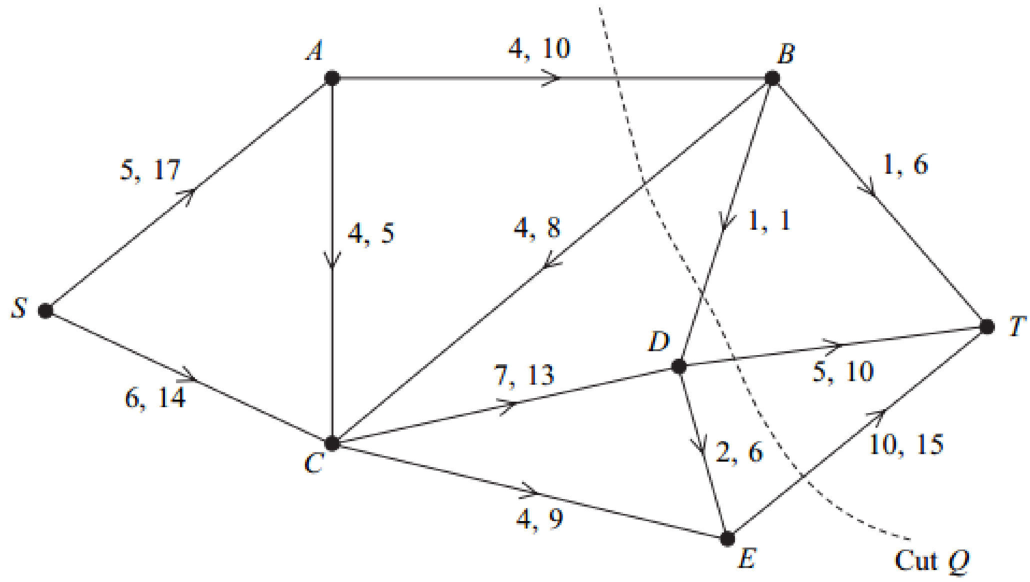
6 The network shows a system of pipes with the lower and upper capacities for each pipe in litres per minute.



(a) Find the value of the cut Q. *(2 marks)*

(b) **Figure 3** opposite shows a partially completed diagram for a feasible flow of 24 litres per minute from S to T. Indicate, on **Figure 3**, the flows along the edges BT, DE and ET. *(2 marks)*

(c) (i) Taking your answer from part **(b)** as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 4** opposite. *(2 marks)*

(ii) Use flow augmentation on **Figure 4** to find the maximum flow from S to T.

You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

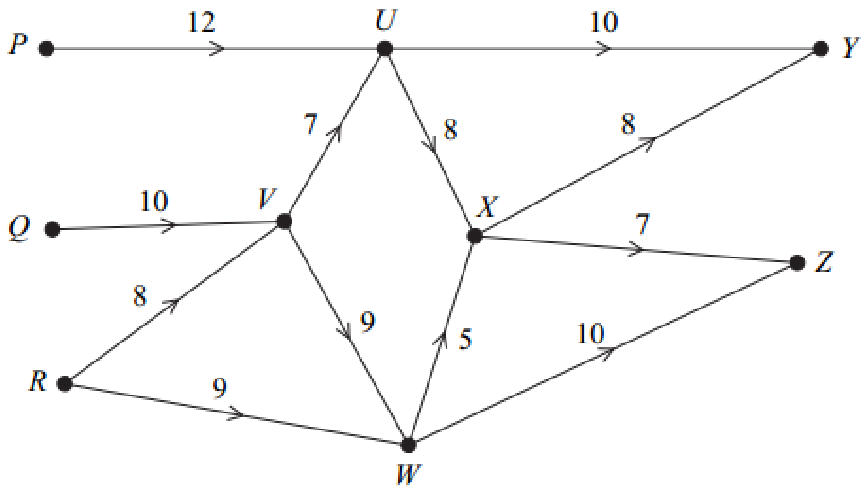
(iii) Illustrate the maximum flow on **Figure 5** opposite. *(2 marks)*

(d) Find a cut with value equal to that of the maximum flow.

You may wish to show the cut on the network above. *(1 mark)*

6 A retail company has warehouses at P, Q and R, and goods are to be transported to retail outlets at Y and Z. There are also retaining depots at U, V, W and X.

The possible routes with the capacities along each edge, in van loads per week, are shown in the following diagram.



(a) On **Figure 5 opposite**, add a super-source, S, and a super-sink, T, and appropriate edges so as to produce a directed network with a single source and a single sink. Indicate the capacity of each edge that you have added. *(2 marks)*

(b) On **Figure 6**, write down the maximum flows along the routes $SPUYT$ and $SRVWZT$. *(2 marks)*

(c) (i) On **Figure 7**, add the vertices S and T and the edges connecting S and T to the network. Using the maximum flows along the routes $SPUYT$ and $SRVWZT$ found in part **(b)** as the initial flow, indicate the potential increases and decreases of the flow on each edge of these routes. *(2 marks)*

(ii) Use flow augmentation to find the maximum flow from S to T. You should indicate any flow-augmenting routes on **Figure 6** and modify the potential increases and decreases of the flow on **Figure 7**. *(4 marks)*

(d) Find a cut with value equal to the maximum flow. *(1 mark)*

JAN 2011 – FIGURE 5, 6 & 7

Figure 5

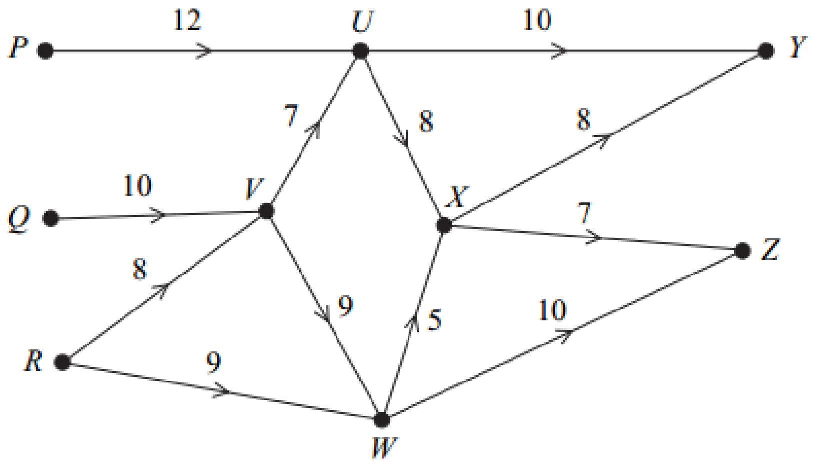
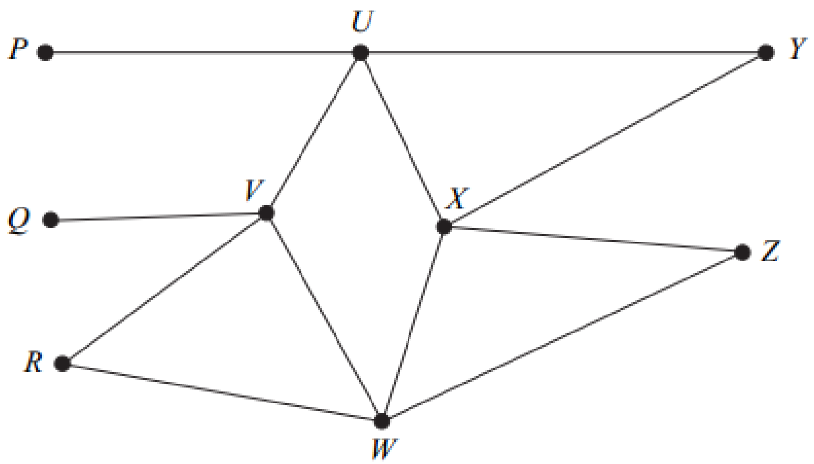


Figure 6

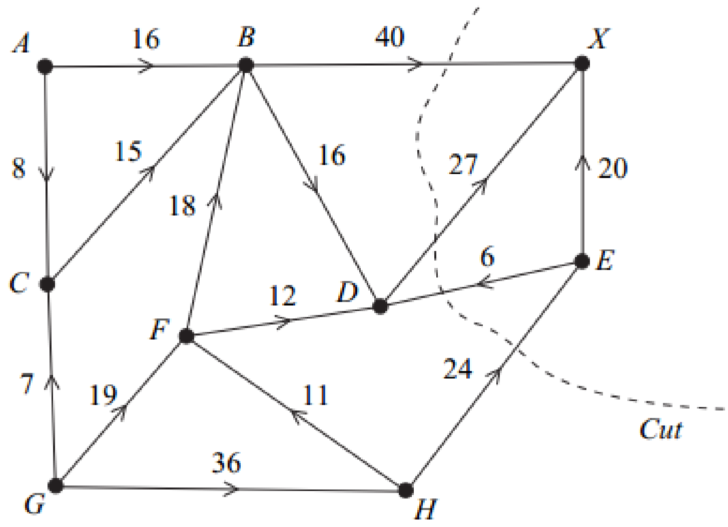
Route	Flow
SPUYT	
SRVWZT	

Figure 7



JUNE 2011

5 The network shows the evacuation routes along corridors in a college, from two teaching areas to the exit, in case of a fire alarm sounding.



The two teaching areas are at A and G and the exit is at X.

The number on each edge represents the maximum number of people that can travel along a particular corridor in one minute.

(a) Find the value of the cut shown on the diagram. *(1 mark)*

(b) Find the maximum flow along each of the routes $ABDX$, $GFBX$ and $GHEX$ and enter their values in the table on **Figure 3** opposite. *(3 marks)*

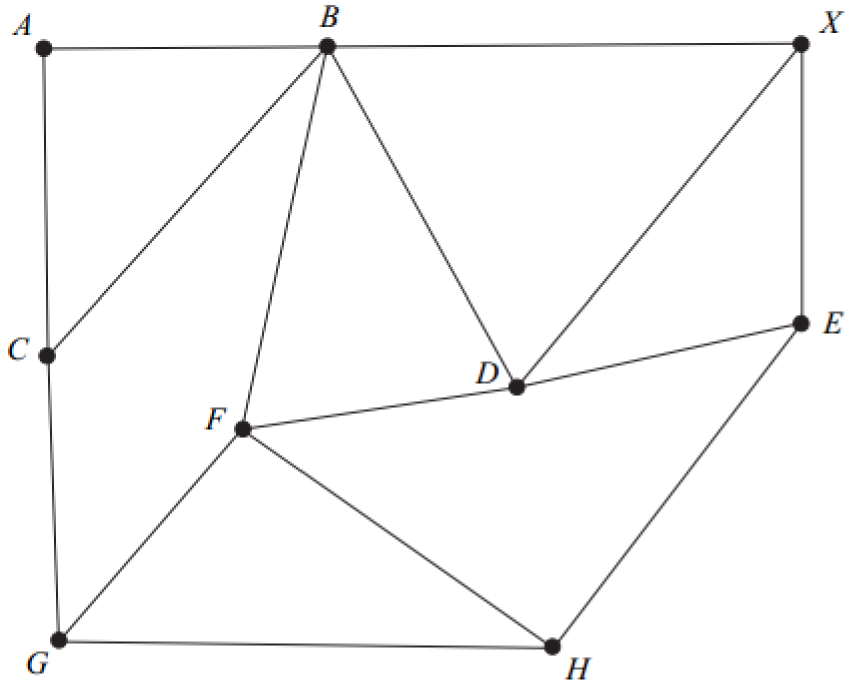
(c) (i) Taking your answers to part **(b)** as the initial flow, use the labelling procedure on **Figure 3** to find the maximum flow through the network. You should indicate any flow augmenting routes in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(ii) State the value of the maximum flow, and, on **Figure 4**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(d) During one particular fire drill, there is an obstruction allowing no more than 45 people per minute to pass through vertex B. State the maximum number of people that can move through the network per minute during this fire drill. *(2 marks)*

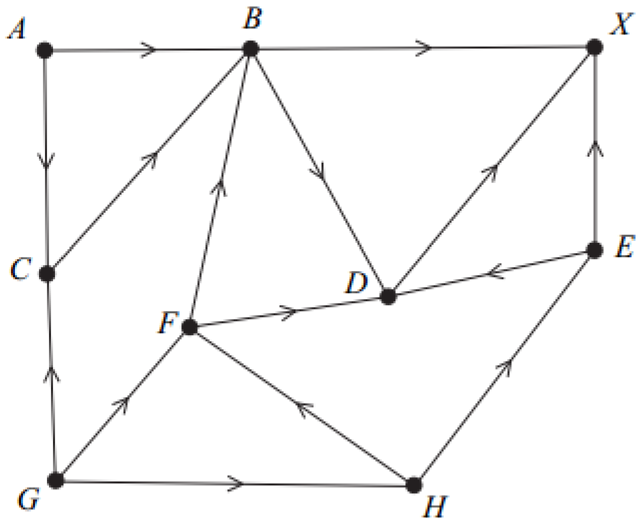
JUNE 2011 – FIGURE 3 & 4

Figure 3



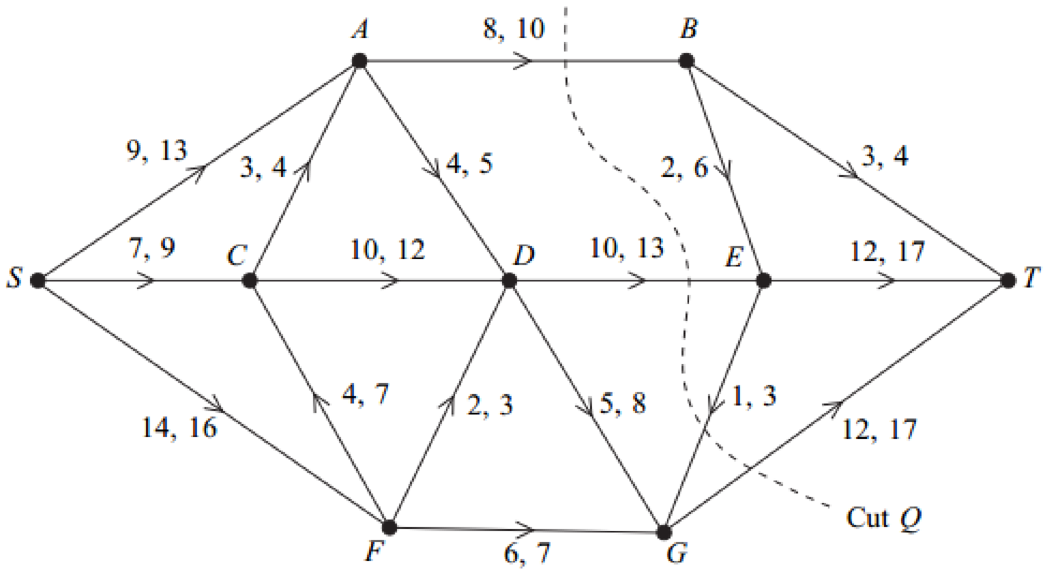
Route	Flow
ABDX	
GFBX	
GHEX	

Figure 4



Maximum flow is .... **people per minute.**

6 The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



(a) Find the value of the cut Q. *(2 marks)*

(b) **Figure 2** shows most of the values of a feasible flow of 34 litres per second from S to T.

(i) Insert the missing values of the flows along DE and FG on **Figure 2**. *(2 marks)*

(ii) Using this feasible flow as the initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 3**. *(2 marks)*

(iii) Use flow augmentation on **Figure 3** to find the maximum flow from S to T. You should indicate any flow-augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(4 marks)*

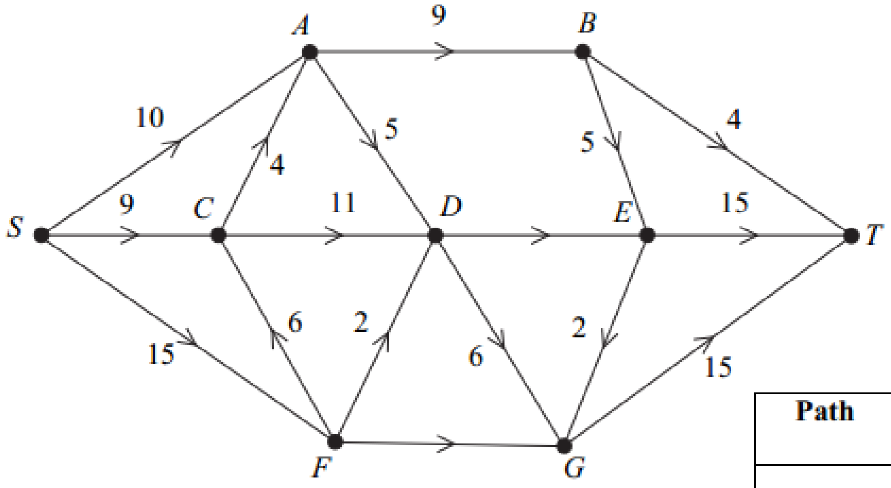
(c) (i) State the value of the maximum flow. *(1 mark)*

(ii) Illustrate your maximum flow on **Figure 4**. *(2 marks)*

(d) Find a cut with capacity equal to that of the maximum flow. *(1 mark)*

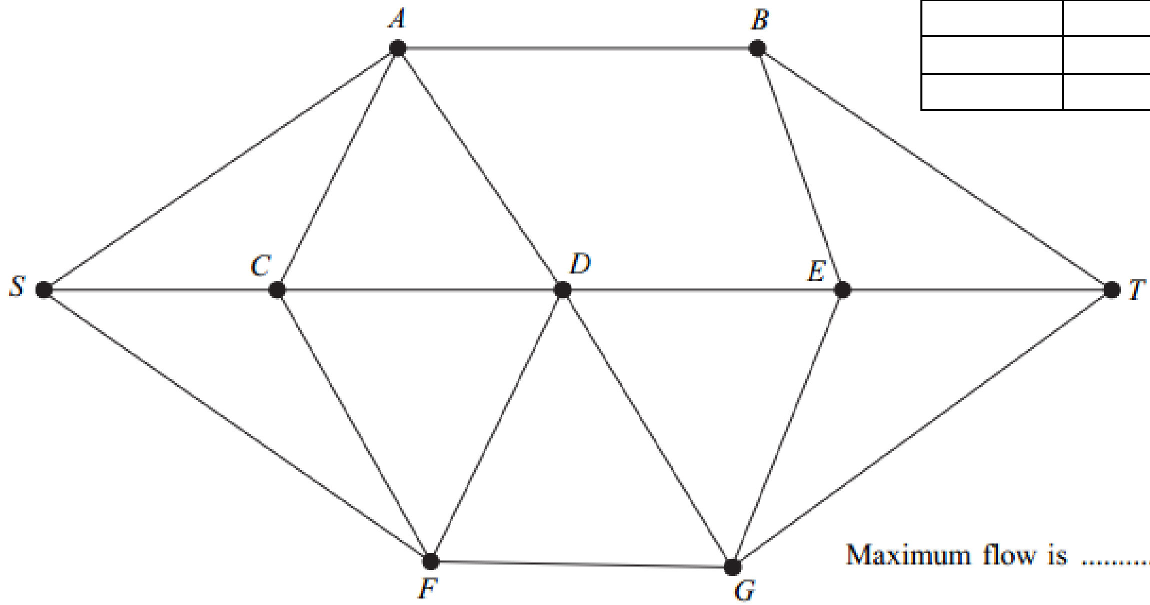
JAN 2012 – FIGURES 2, 3 & 4

Figure 2



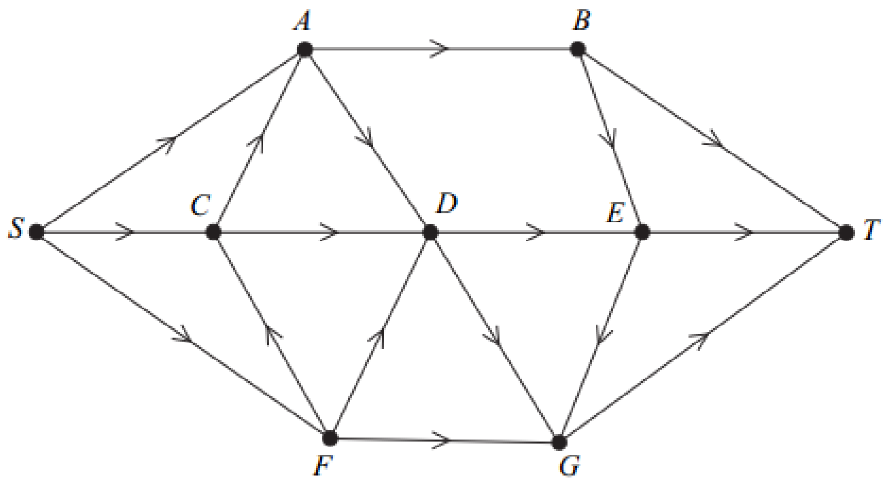
Path	Extra Flow

Figure 3



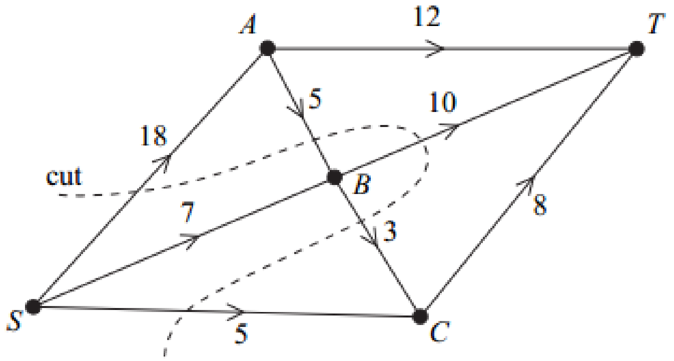
Maximum flow is ....

Figure 4



JUNE 2012

6 (a) The network shows a flow from S to T along a system of pipes, with the capacity in litres per second indicated on each edge.



(i) Show that the value of the cut shown on the diagram is 36. *(1 mark)*

....

....

(ii) The cut shown on the diagram can be represented as $\{S, B\}$, $\{A, C, T\}$.

Complete the table below to give the value of each of the 8 possible cuts. *(3 marks)*

Cut		Value
$\{S\}$	$\{A, B, C, T\}$	30
$\{S, A\}$	$\{B, C, T\}$	29
$\{S, B\}$	$\{A, C, T\}$	36
$\{S, C\}$	$\{A, B, T\}$	33
$\{S, A, B\}$	$\{C, T\}$	
$\{S, A, C\}$	$\{B, T\}$	
$\{S, B, C\}$	$\{A, T\}$	
$\{S, A, B, C\}$	$\{T\}$	30

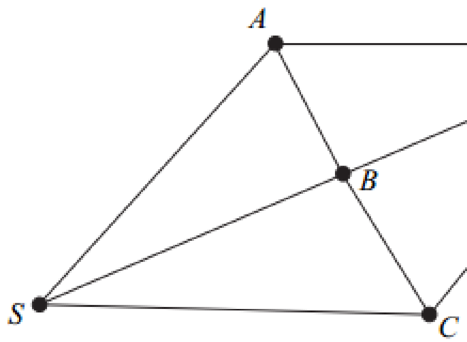
(iii) State the value of the maximum flow through the network, giving a reason for your answer. *(2 marks)*

Maximum flow is ....

because ....

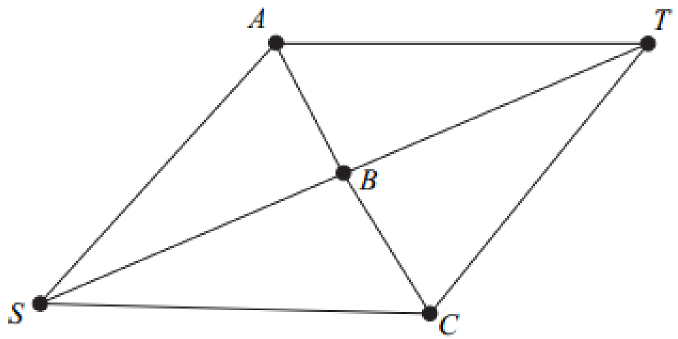
....

(iv) Indicate on the diagram below a possible flow along each edge corresponding to this maximum flow. *(1 mark)*



(b) The capacities along SC and along AT are each increased by 4 litres per second.

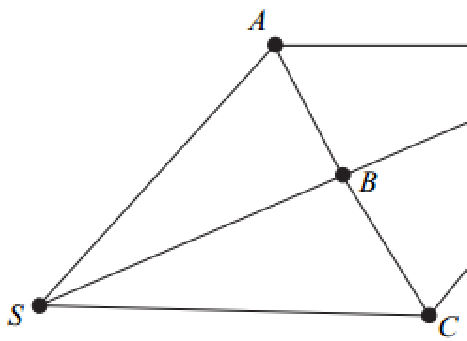
(i) Using your **values from part (a)(iv) as the initial flow**, indicate potential increases and decreases on the diagram below and use the labelling procedure to find the new maximum flow through the network. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the diagram. *(6 marks)*



Path	Additional Flow

(ii) Use your results from part **(b)(i)** to illustrate the flow along each edge that gives this new maximum flow, and state the value of the new maximum flow. *(3 marks)*

New maximum flow is ....



JAN 2013

4 (a) When investigating three network flow problems, a student finds:

(i) a flow of 50 and a cut with capacity 50;

(ii) a flow of 35 and a cut with capacity 50;

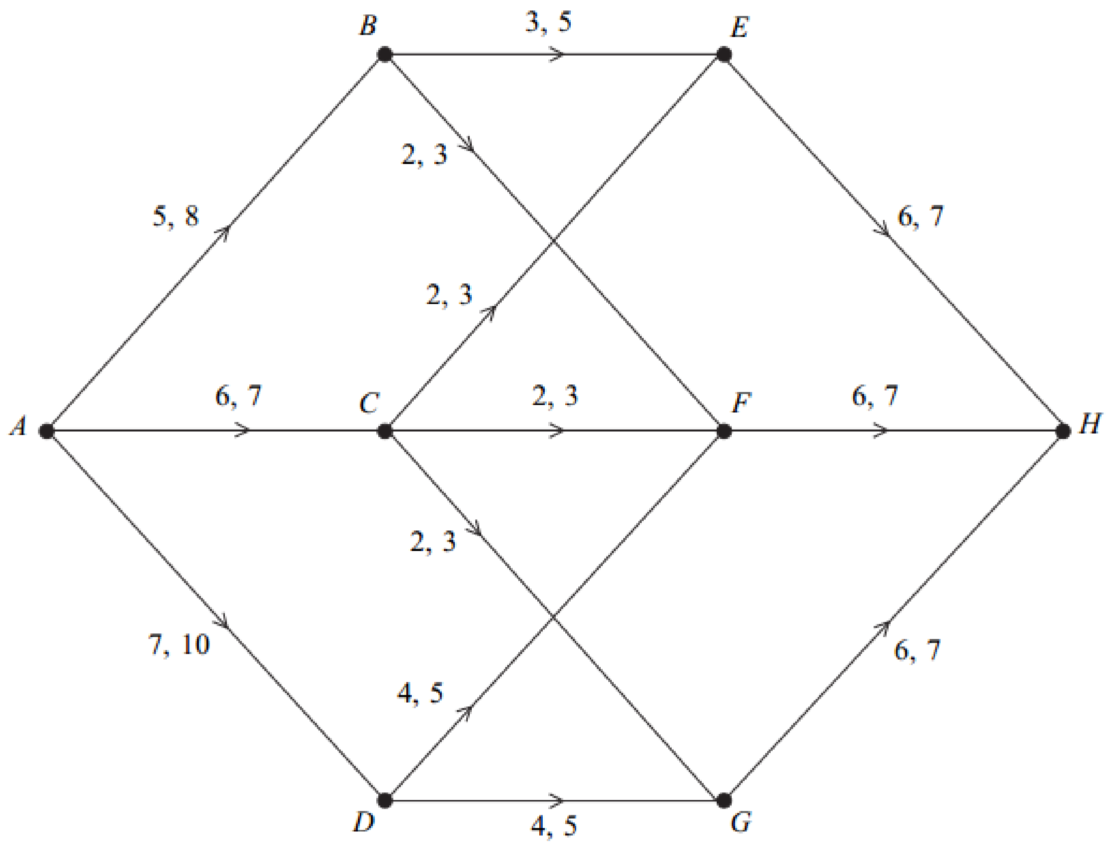
(iii) a flow of 50 and a cut with capacity 35.

In each case, write down what the student can deduce about the maximum flow.

(4 marks)

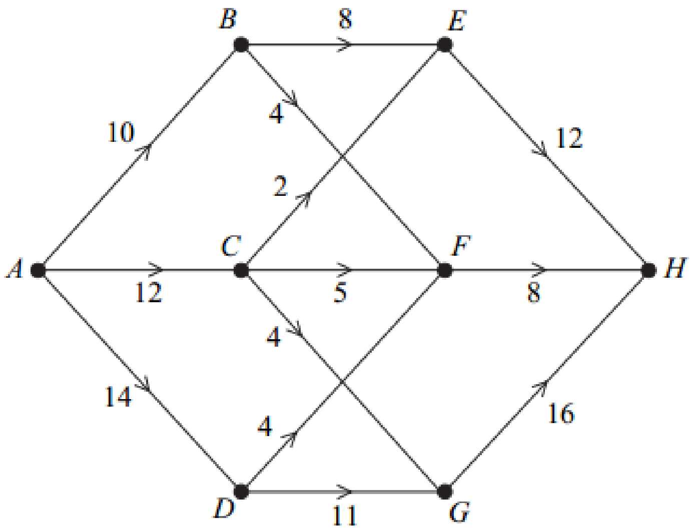
(b) The diagram below shows a network. The numbers on the arcs represent the minimum and maximum flow along each arc respectively.

By considering the flow at an appropriate vertex, explain why a flow is not possible through this network.



(2 marks)

8 The network below represents a system of pipes. The capacity of each pipe, in litres per second, is indicated on the corresponding edge.



(a) Find the maximum flow along each of the routes *ABEH*, *ACFH* and *ADGH* and enter their values in the table on **Figure 4** opposite. *(1 mark)*

(b) (i) Taking your answers to part **(a)** as the initial flow, use the labelling procedure on **Figure 4** to find the maximum flow through the network. You should indicate any flow-augmenting routes in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(ii) State the value of the maximum flow and, on **Figure 5** opposite, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(c) Confirm that you have a maximum flow by finding a cut of the same value. List the edges of your cut. *(1 mark)*

JAN 2013 – Q8 FIGURES 4 & 5

Answer space for question 8

Route	Flow
ABEH	
ACFH	
ADGH	

Figure 4

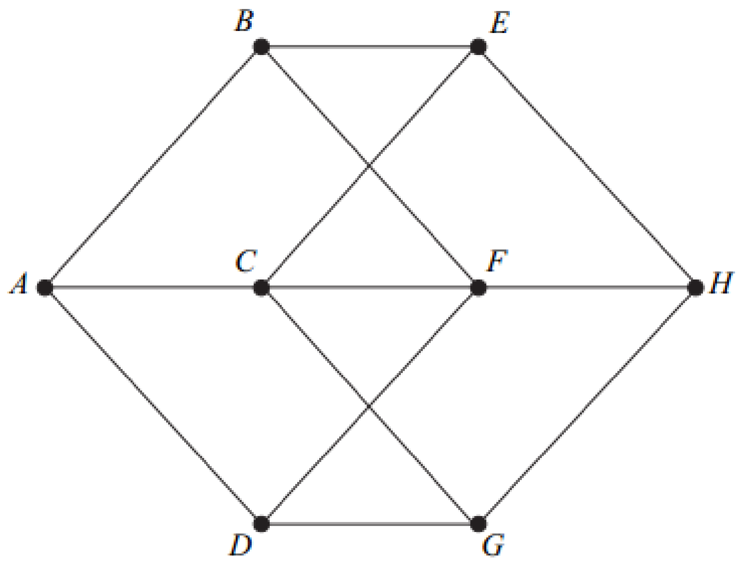
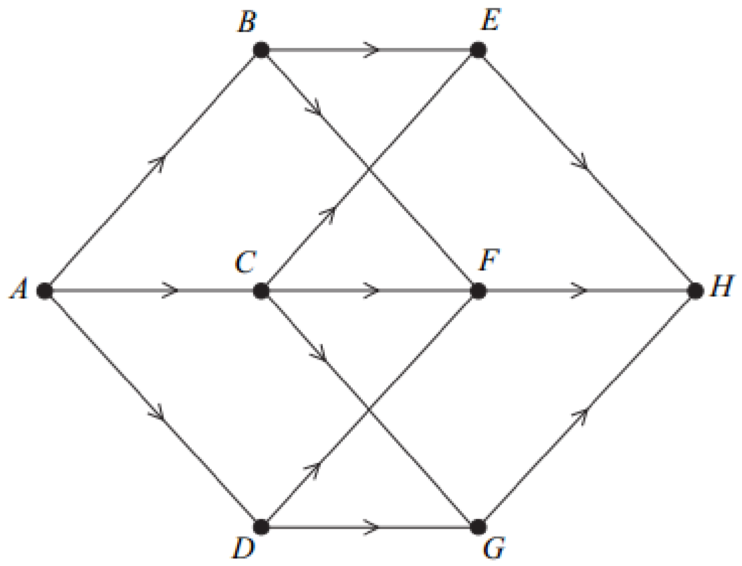
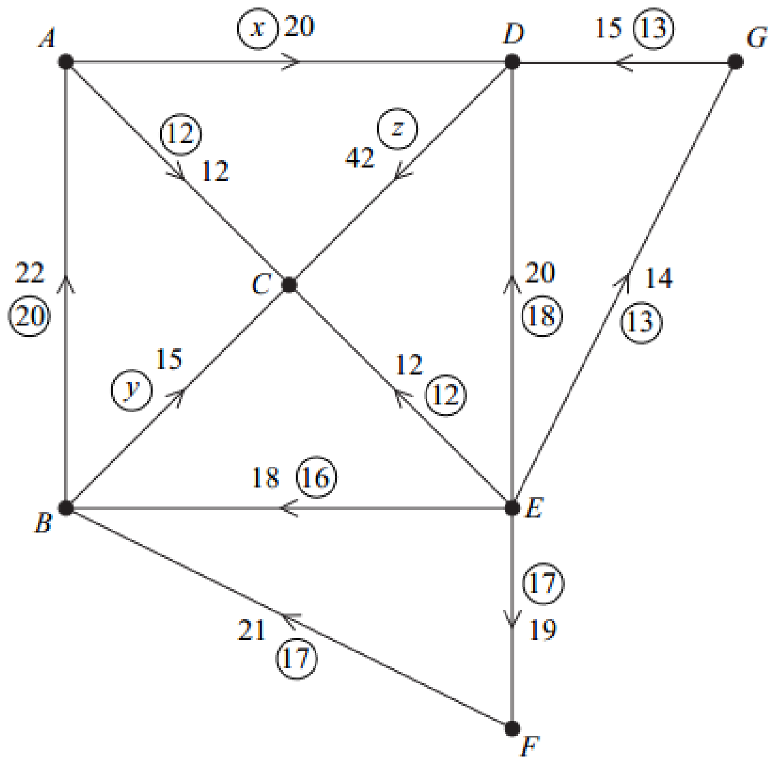


Figure 5



JUNE 2013

2 The network below represents a system of pipes. The number **not** circled on each edge represents the capacity of each pipe in litres per second. The number or letter in each circle represents an initial flow in litres per second.



(a) Write down the capacity of edge EF. *(1 mark)*

(b) State the source vertex. *(1 mark)*

(c) State the sink vertex. *(1 mark)*

(d) Find the values of x, y and z. *(3 marks)*

(e) Find the value of the initial flow. *(1 mark)*

(f) Find the value of a cut through the edges EB, EC, ED, EF and EG. *(1 mark)*

7 **Figure 2** shows a network of pipes.

Water from two reservoirs, R_1 and R_2, is used to supply three towns, T_1, T_2 and T_3.

In **Figure 2**, the capacity of each pipe is given by the number **not** circled on each edge. The numbers in circles represent an initial flow.

(a) Add a supersource, supersink and appropriate weighted edges to **Figure 2**. *(2 marks)*

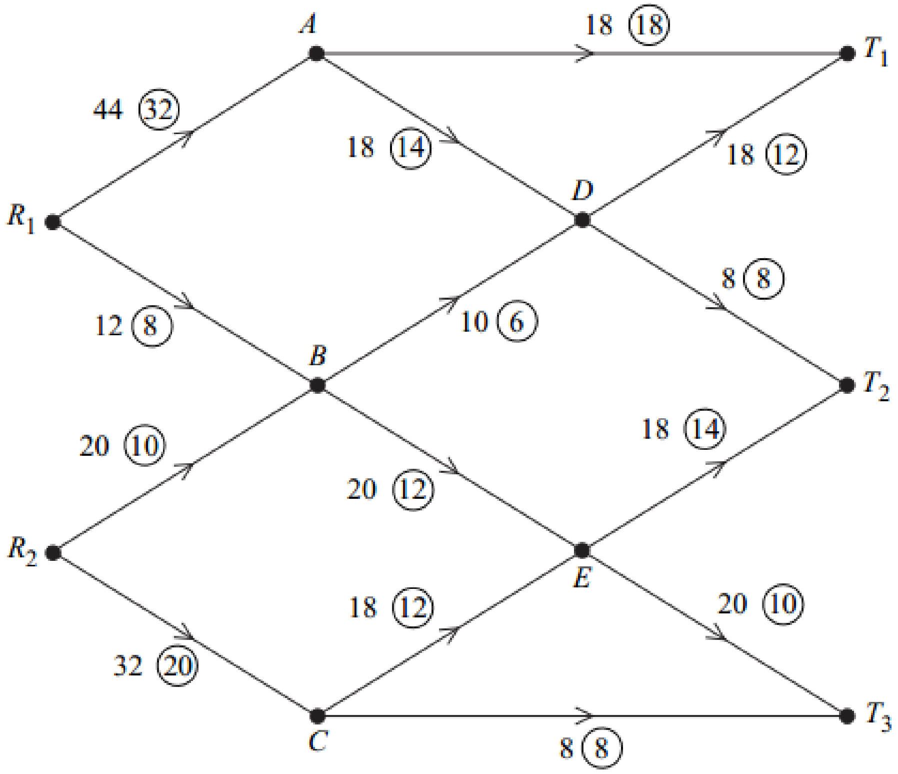
(b) (i) Use the initial flow and the labelling procedure on **Figure 3** to find the maximum flow through the network.

You should indicate any flow augmenting routes in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(ii) State the value of the maximum flow and, on **Figure 4**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(c) Confirm that you have a maximum flow by finding a cut of the same value. List the edges of your cut. *(2 marks)*

Figure 2



JUNE 2013 – Q7 FIGURES 3 & 4

Figure 3

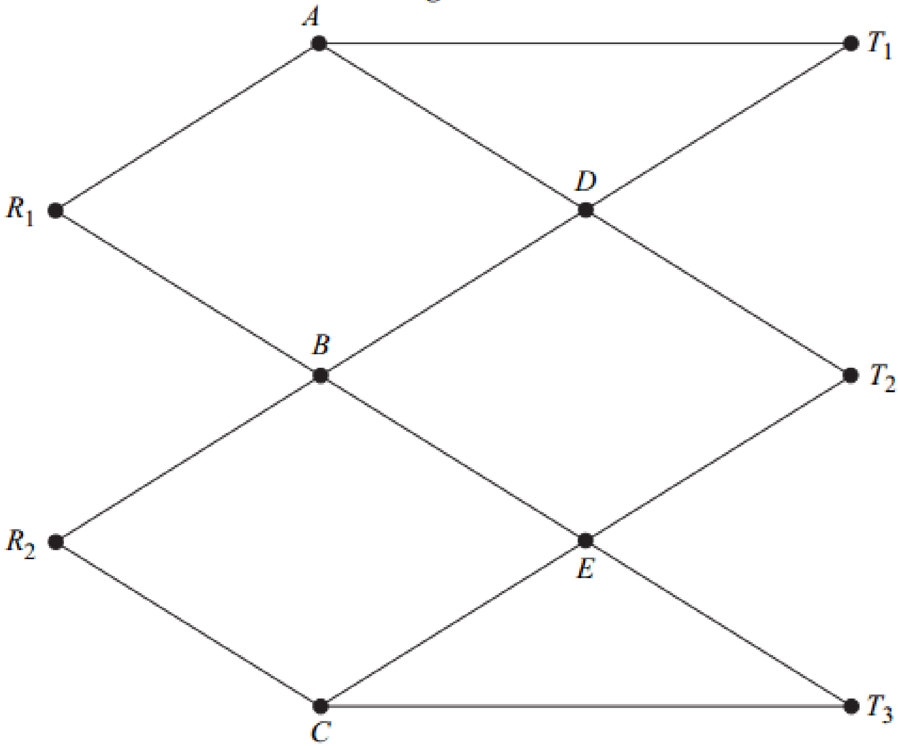
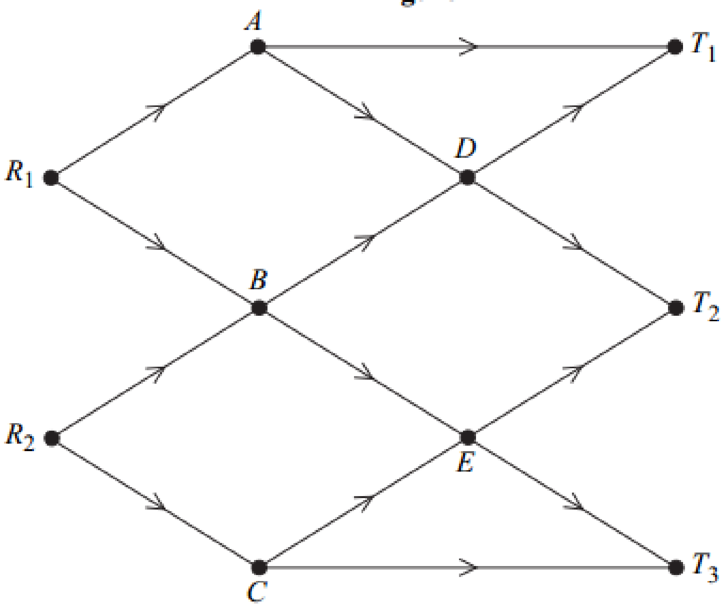
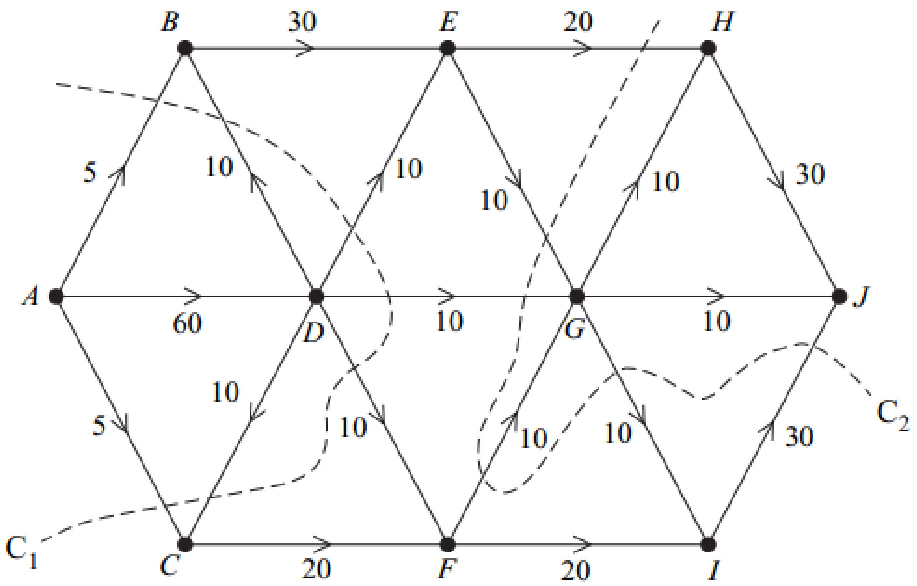


Figure 4



Route	Flow

3 The diagram below shows a network of pipes with source A and sink J. The capacity of each pipe is given by the number on each edge.



(a) Find the values of the cuts C_1 and C_2.

[2 marks]

(b) Find by inspection a flow of 60 units, with flows of 25, 10 and 25 along HJ, GJ and IJ respectively. Illustrate your answer on **Figure 1**.

[2 marks]

(c) (i) On a certain day the section EH is blocked, as shown on **Figure 2**.

Find, by inspection or otherwise, the maximum flow on this day and illustrate your answer on **Figure 2**.

[3 marks]

(ii) Show that the flow obtained in part **(c)(i)** is maximal.

[2 marks]

JUNE 2014 – FIGURES 1 & 2

Answer space for question 3

Figure 1

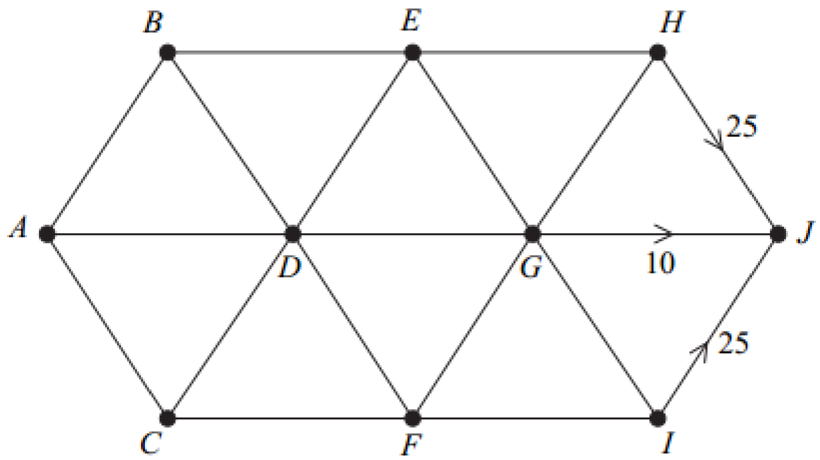
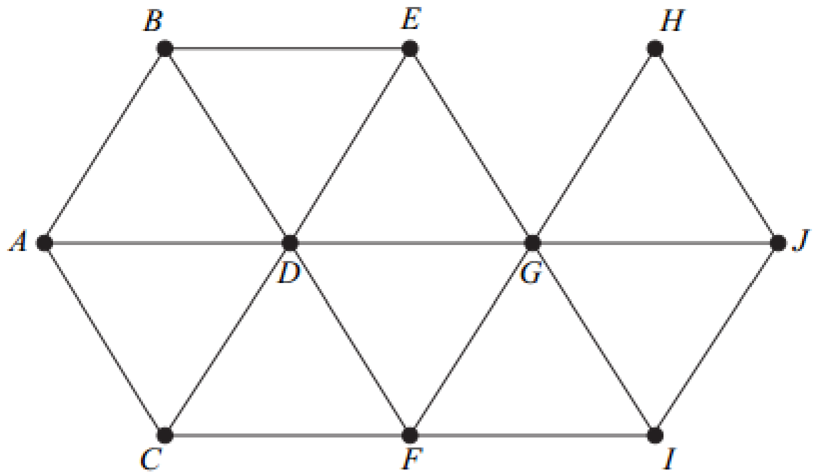


Figure 2



Maximum flow = ______________________________